Cáp. 05 ⇒ A Reta.   (1)

$\underline{01)}$ $P = A + t(B-A)$

$P(x,y,z)$ | I) $(x,y,z) = (2,-3,4) + t(-1,2,-2)$

$A(2,-3,4)$ |

$B(1,-1,2)$ |

II) $C(5/2, -4, 5)$ III) $D(-1,3,4)$

$\begin{cases} x = 2+t \\ y = -3+2t \\ z = 4-2t \end{cases}$ $\begin{aligned} 5/2 &= 2-t \\ t &= -1/2 \\ -4 &= -3+2t \end{aligned}$ $\begin{aligned} x &= 2-t & y &= -3+2t & z &= 4-2t \\ -1 &= 2-t & 3 &= -3+2t & 4 &= 4-2t \\ t &= 3 & \underline{t=3} & -2t &= 0 \end{aligned}$

$5 = 4-2t$ $t = -1/2$ $t = 0$

$\underline{t = -1/2}$ $D \notin r$

         $C \in r$

$\underline{02)}$ $r: (x,y,z) = (-1,2,3) + t(2,-3,0)$

$r: \begin{cases} x = -1+2t \\ y = 2-3t \\ z = 3 \end{cases}$

$\underline{03)}$ $\vec{v} = (0,0,1)$

$r: (x,y,z) = (1,4,3) + t(0,0,1)$ $r: \begin{cases} x = 1 \\ y = 2 \\ z = 3+t \end{cases}$

$A(1,2,3)$

$\underline{04)}$ $r: \begin{cases} x = 2+t \\ y = 3-t \\ z = -4+2t \end{cases}$ a) $y = 6$ $6 = 3-t$ $x = 2+t$ $z = -4+2(3)$

                      $P(x,6,z)$ $\underline{t = -3}$ $\underline{x = -1}$ $z = -10$

                                                      $\boxed{P(-1,6,-10)}$

b) $\underline{x=y}$ $2+t = 3-t$ $x = 2+1/2 = 5/2$

$Q(x,y,z)$ $2t = 1$ $y = 3 - 1/2 = 5/2$ $\boxed{Q(5/2, 5/2, -3)}$

                $t = 1/2$ $z = -4 + 2(1/2) = -3$

c) $z = 4x$ $-4 + 2t = 4(2+t)$ $x = 2 - 6 = -4$

$R(x,y,z)$ $-4 + 2t = 8 + 4t$ $y = 3 - (-6) = 9$

$2t = -12$ $z = -4 + 2(-6) = -16$

$t = -6$

$\{R(-4, 9, -16)\}$

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05) $A(4, -3, -2)$ $S: (x,y,z) = (1, 2, 3) + t(3, -4, -1)$

$S: \begin{cases} x = 1 + 3t \\ y = 2 - 4t \\ z = 3 - t \end{cases}$ $M: (m, n, -5) = (4, -3, -2) + t(3, -4, -1)$

$-5 = -2 - t$ $m = 4 + 3t$ $n = -3 - 4t$

obs: $t = 3$ $m = 4 + 3 \cdot 3$ $n = -3 - 4 \cdot 3$

como s//r, os vetores $m = 13$ $n = -15$

diretos são iguais.

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06) a) $P = A + t(B-A)$ c) $(x,y,z) = (1, 2, 3) + t(0, 1, -1)$

$(x,y,z) = (1, -1, 2) + t(1, 2, -2)$ $M: \begin{cases} x = 1 \\ y = 2 + t \\ z = 3 - t \end{cases}$

$M: \begin{cases} x = 1 + t \\ y = -1 + 2t \\ z = 2 - 2t \end{cases}$

b) $(x,y,z) = (3, 1, 4) + t(0, -3, -2)$ d) $(x,y,z) = (0,0,0) + t(0, 1, 0)$

$M: \begin{cases} x = 3 \\ y = 1 - 3t \\ z = 4 - 2t \end{cases}$ $M: \begin{cases} x = 0 \\ y = t \\ z = 0 \end{cases}$

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07) a) $A(2, 0, 4)$ | $\{P = A + t(B-A)\}$ $(x,y,z) = (2, 0, 4) + t(-2, 0, 0)$

$B(0, 0, 4) =$

$P(x,y,z)$ $M: \begin{cases} x = 2 - 2t \\ y = 0 \\ z = 4 \end{cases}$

b) $C(0,3,0)$

$P = C + t(D-C)$

$(x,y,z) = (0,3,0) + t(2,0,0)$

$D(2,3,0)$

$r: \begin{cases} x = 2t \\ y = 3 \\ z = 0 \end{cases}$

$P(x,y,z)$

c) $A(2,0,4)$

$P = A + t(D-A)$

$(x,y,z) = (2,0,4) + t(0,3,-4)$

$D(2,3,0)$

$r: \begin{cases} x = 2 \\ y = 3t \\ z = 4-4t \end{cases}$

$P(x,y,z)$

d) $B(0,0,4)$

$P = B + t(C-B)$

$(x,y,z) = (0,0,4) + t(0,3,-4)$

$C(0,3,0)$

$r: \begin{cases} x = 0 \\ y = 3t \\ z = 4-4t \end{cases}$

$P(x,y,z)$

e) $D(2,3,0)$

$P = D + t(E-D)$

$(x,y,z) = (2,3,0) + t(0,-3,0)$

$E(2,0,0)$

$r: \begin{cases} x = 2 \\ y = 3-3t \\ z = 0 \end{cases}$

$P(x,y,z)$

f) $B(0,0,4)$

$P = B + t(D-B)$

$(x,y,z) = (0,0,4) + t(2,3,-4)$

$D(2,3,0)$

$r: \begin{cases} x = 2t \\ y = 3t \\ z = 4-4t \end{cases}$

$P(x,y,z)$

8) $P(m,1,n)$

$r: P = A + t(B-A)$

$(m,1,n) = (3,-1,4) + t(1,-2,-5)$

$A(3,-1,4)$

$B(4,-3,-1)$

$r: \begin{cases} m = 3+t \\ 1 = -1-2t \\ n = 4-5t \end{cases}$

$1 = -1-2t$	$m = 3+t$	$n = 4-5t$
$-2t = 2$	$m = 3-1$	$n = 4-5(-1)$
$t = -1$	$m = 2$	$n = 4+5$
		$n = 9$

$\boxed{P(2,1,9)}$

09)
R.

$C(2,-1,4)$

$A(-1,4,-2)$ M $B(3,-3,6)$
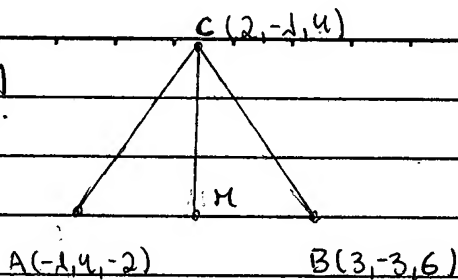
$$M = \left(\frac{x_A + x_B}{2}, \frac{x_A + y_B}{2}, \frac{z_A + z_B}{2} \right)$$

$$M = \left(\frac{-1+3}{2}, \frac{4+(-3)}{2}, \frac{-2+6}{2} \right)$$
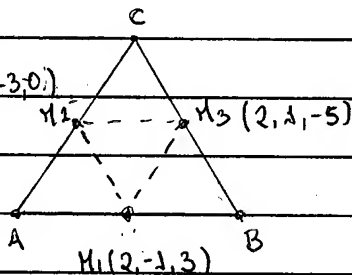
$$M = (1, 1/2, 2)$$

$\vec{MC} = (1, -3/2, 2)$

$P = M + t(C - M)$

$(x,y,z) = (1, 1/2, 2) + t(1, -3/2, -2)$

$$r: \begin{cases} x = 1 + t \\ y = 1/2 - 3/2\,t \\ z = 2 + 2t \end{cases}$$

10) $(1,-3,0)$
R.

C

M_2 - - - - M_3 $(2,1,-5)$

A $M_1(2,-1,3)$ B

$\vec{M_2 M_3} = \frac{1}{2}(\vec{AC} + \vec{CB})$

$\vec{M_2 M_3} = \frac{1}{2}\vec{AB} \implies \vec{M_2 M_3} = (1,4,-5)$!

$P = M_1 + t(M_3 - M_2)$

$(x,y,z) = (2,-1,3) + t(1,4,-5)$

$$r: \begin{cases} x = 2 + t \\ y = -1 + 4t \\ z = 3 - 5t \end{cases}$$

11)
R.

C $(3,-1,-1)$

$(1,0,1)$ M

$A (-1,1,3)$ $B (2,1,4)$
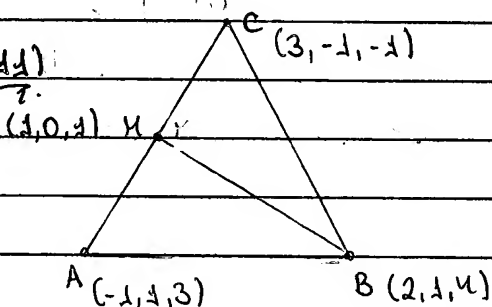
I) $\vec{AB} = \vec{v}$

$P = A + t(B - A)$

$(x,y,z) = (-1,1,3) + t(3,0,1)$

$$r: \begin{cases} x = -1 + 3t \\ y = 1 \\ z = 3 + t \end{cases}$$

II) $\vec{v} = \vec{AC}$ $P = A + t(C - A)$

$(x,y,z) = (-1,1,3) + t(4,-2,-4)$

$$r: \begin{cases} x = -1 + 4t \\ y = 1 - 2t \\ z = 3 - 4t \end{cases}$$

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III) $\vec{v} = \vec{BC}$ $\quad\quad P = B + t(C-B)$

$P(x,y,z)$ $\quad\quad (x,y,z) = (2,1,4) + t(1,-2,-5)$

$$t: \begin{cases} x = 2+t \\ y = 1-2t \\ z = 4-5t \end{cases}$$

IV) $\vec{MB} = \vec{v}$ $\quad\quad P = M + t(B-M)$

$P(x,y,z)$ $\quad\quad (x,y,z) = (1,0,1) + t(-1,1,3)$

$$M: \begin{cases} x = 1+t \\ y = t \\ z = 1+8t \end{cases}$$

$\underset{3.}{12)}$ $\quad r: \dfrac{x-3}{-1} = \dfrac{y+1}{2} = \dfrac{z-2}{-2}$ $\quad \Rightarrow \quad r:(x,y,z) = (3,-1,2) + t(-1,2,-2)$

$$\Rightarrow \quad r: \begin{cases} x = 3-t \\ y = -1+2t \\ z = 2-2t \end{cases}$$

I) $P_1(5,-5,6)$

$5 = 3-t$ $\quad\quad -5 = -1+2t$ $\quad\quad 6 = 2-2t$

$\underline{t = -2}$ $\quad\quad \underline{t = -2}$ $\quad\quad \underline{t = -2}$ $\quad\quad$ ⟮$P_1 \in r$⟯

II) $P_2(4,-1,12)$

$4 = 3-t$ $\quad\quad -1 = -1+2t$ $\quad\quad 12 = 2-2t$ $\quad\quad$ ⟮$P_2 \notin r$⟯

$\underline{t = -1}$ $\quad\quad \underline{t = 0}$ $\quad\quad \underline{t = -5}$

$\underset{3.}{13)}$ $\quad r:(x,y,z) = (1,-3,0) + t(2,-1,4)$

$$\Rightarrow r: \begin{cases} x = 1+2t \\ y = -3-t \\ z = 4t \end{cases}$$

a) $x = 5$; $\quad 5 = 1+2t$ $\quad\quad y = -3-t$

$\quad\quad\quad\quad\quad 2t = 4$ $\quad\quad\quad y = -3-2$

$\quad\quad\quad\quad\quad \underline{t = 2}$ $\quad\quad\quad \underline{y = -5}$

$z = 4t$

$z = 4\cdot 2$ $\quad\quad$ ⟮$P(5,-5,8)$⟯

$\underline{z = 8}$

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b) $y = 2$;

$y = -3 - t$　　　$x = 1 + 2t$　　　$z = 4t$

$2 = -3 - t$　　　$x = 1 + 2(-5)$　　　$z = 4(-5)$

$t = -5$　　　$x = -9$　　　$z = -20$

$Q(-9, 2, -20)$

14) $P(1, y, z)$　　$r: (2x, 3y, z) = (-1, 2, -4) + t(3, 2, 1)$
3.

$2 = -1 + 3t$　　$3y = 2 + 2t$　　$r: \begin{cases} 2x = -1 + 3t \\ 3y = 2 + 2t \\ z = -4 + t \end{cases}$

$t = 1$　　$3y = 2 + 2$

　　　　$y = 4/3$　　　$P(1, 4/3, -3)$

$z = -4 + 1$

$z = -3$

5) vetor diretor $(x, 2, z)$

$r: \dfrac{2x + 1}{3} = \dfrac{3y - 2}{2} = \dfrac{z + 4}{1}$　$\Rightarrow$　$\dfrac{x + \frac{1}{2}}{3/2} = \dfrac{y - \frac{2}{3}}{2/3} = \dfrac{z + 4}{1}$

$r: (x, y, z) = (-1/2, 2/3, -4) + t(3/2, 2/3, 1)$

　　　　　　　　　$\downarrow$

　　　　　　　vetor diretor

　　　　　　$3 \cdot (3/2, 2/3, 1)$

　　　　　　$(9/2, 2, 3)$

15) a) $A(4,0,-3)$ $\eta: (x,y,z) = (4,0,-3) + t(2,4,5)$
7. $\vec{v} = (2,4,5)$

$$\Rightarrow \eta: \frac{x-4}{2} = \frac{y}{4} = \frac{z+3}{5}$$

$$\Rightarrow \eta: \begin{cases} y = 2x - 8 \\ z = 5x/2 - 13 \end{cases}$$

b) $A(1,-2,3)$ $P = A + t(B-A)$

$B(3,-1,-1)$ $\eta: (x,y,z) = (1,-2,3) + t(2,1,-4)$

$P(x,y,z)$

$$\Rightarrow \eta: \frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{-4}$$

$\frac{x-1}{2} = \frac{y+2}{1}$ $\frac{x-1}{2} = \frac{z-3}{-4}$ $\Rightarrow \eta: \begin{cases} y = \dfrac{x-5}{2} \\ \\ z = -2x + 5 \end{cases}$

$\frac{x-1}{2} = y+2$ $z - 3 = -2x + 2$

                 $\underline{z = -2x + 5}$

$y = \dfrac{x-1-4}{2} = \dfrac{x-5}{2}$

c) $A(-1,2,3)$ $P = A + t(B-A)$

$B(2,-1,3)$ $\eta: (x,y,z) = (-1,2,3) + t(3,-3,0)$

$P(x,y,z)$ $\Rightarrow \eta: \dfrac{x+1}{3} = \dfrac{y-2}{-3} = \dfrac{z-3}{0}$

$\frac{x+1}{3} = \frac{y-2}{-3}$ $\frac{x+1}{3} = \frac{z-3}{0}$ $\Rightarrow \eta: \begin{cases} y = -x + 1 \\ z = 3 \end{cases}$

$y - 2 = -x - 1$ $z - 3 = 0$

$\underline{y = -x + 1}$ $\underline{z = 3}$

d) $M: \begin{cases} x = 2 - t \\ y = 3t \\ z = 4t - 5 \end{cases}$ $\Rightarrow M: \dfrac{x-2}{-1} = \dfrac{y}{3} = \dfrac{z+5}{4}$

Equações reduzidas:

$\dfrac{x-2}{-1} = \dfrac{y}{3}$ $\dfrac{x-2}{-1} = \dfrac{z+5}{4}$ $\Rightarrow M: \begin{cases} y = -3x + 6 \\ z = -4x + 3 \end{cases}$

$\underline{y = -3x + 6}$ $z + 5 = -4x + 8$

$\underline{z = -4x + 3}$

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16) $A(-1, 6, 3)$ | $P = A + t(B - A)$

   $B(2, 2, 1)$: $M: (x, y, z) = (-1, 6, 3) + t(3, -4, -2)$

   $P(x, y, z)$ |

$\dfrac{x+1}{3} = \dfrac{z-3}{-2}$ $\dfrac{y-6}{-4} = \dfrac{z-3}{-2}$

Equações Simétricas:

$\Rightarrow M: \dfrac{x+1}{3} = \dfrac{y-6}{-4} = \dfrac{z-3}{-2}$ $x + 1 = \dfrac{-3z + 9}{2}$ $y - 6 = 2z - 6$

    $\underline{y = 2z}$

$x = \dfrac{-3z + 7}{2}$

Equações reduzidas:

$\Rightarrow M: \begin{cases} x = \dfrac{-3z}{2} + \dfrac{7}{2} \\ \\ y = 2z \end{cases}$

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17) $M: \begin{cases} y = 2x + 3 \\ z = x - 1 \end{cases}$ a) $y = 9;$ $9 = 2x + 3$ $z = 3 - 1$ $P(3, 9, 2)$

                                $2x = 6$ $\underline{z = 2}$

                                $\underline{x = 3}$

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b) $x = 2z;$ $z = x - 1$ $1 = x - 1$ $y = 2 \cdot 2 + 3$ $Q(2, 7, 1)$

                  $z = 2z - 1$ $\underline{x = 2}$ $\underline{y = 7}$

                  $\underline{z = 1}$

c) $y = 3z$;

$$\begin{cases} 3z = 2x + 3 \\ z = x - 1 \end{cases} \qquad 3(x-1) = 2x + 3 \qquad z = 6 - 1$$

$$3x - 3 = 2x + 3 \qquad z = 5$$

$y = 3 \cdot 5 = 15$ $\qquad\qquad\qquad x = 6$

$R(6, 15, 5)$

18) a)

M: $\begin{cases} x = 1 - t \\ y = -1 + 2t \\ z = 2 + t \end{cases}$ $\Rightarrow$ M: $(x, y, z) = (1, -1, 2) + t(-1, 2, 1)$

19) a) $\vec{v} = (1, 0, 0)$ $\qquad$ M: $(x, y, z) = (3, -2, 4) + t(1, 0, 0)$

$A(3, -2, 4)$

$P(x, y, z)$ $\qquad \Rightarrow$ M: $\begin{cases} x = 3 + t \\ y = -2 \\ z = 4 \end{cases}$

b) $\vec{v} = (0, 1, 0)$ $\qquad$ s: $(x, y, z) = (2, 2, 4) + t(0, 1, 0)$

$A(2, 2, 4)$

$\Rightarrow$ s: $\begin{cases} x = 2 \\ y = 2 + t \\ z = 4 \end{cases}$

c) $\vec{v}(0, 0, 1)$ $\qquad$ p: $(x, y, z) = (-2, 3, 4) + t(0, 0, 1)$

$A(-2, 3, 4)$

$\Rightarrow$ p: $\begin{cases} x = -2 \\ y = 3 \\ z = 4 + t \end{cases}$

d) $\vec{v}(3, -2, 0)$ $\qquad$ M: $(x, y, z) = (4, -1, 3) + t(3, -2, 0)$

$A(4, -1, 3)$

$\Rightarrow$ M: $\begin{cases} x = 4 + 3t \\ y = -1 - 2t \\ z = 3 \end{cases}$

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e) $A(3,-1,3)$ $\qquad$ $P = A + t(B-A)$

$B(3,3,4)$ $\qquad$ $r: (x,y,z) = (3,-1,3) + t(0,4,1)$

$P(x,y,z)$

$\Rightarrow r: \begin{cases} x = 3 \\ y = -1 + 4t \\ z = 3 + t \end{cases}$

20) $A(4,-5,3)$ $\qquad$ I) $Ox \Rightarrow \vec{v} = (1,0,0)$

$r: (x,y,z) = (4,-5,3) + t(1,0,0)$

$\Rightarrow r: \begin{cases} x = 4 + t \\ y = -5 \\ z = 3 \end{cases}$

II) $Oy \Rightarrow \vec{v} = (0,1,0)$ $\qquad\qquad$ III) $O_z \Rightarrow \vec{v} = (0,0,1)$

$s: (x,y,z) = (4,-5,3) + t(0,1,0)$ $\qquad$ $p: (x,y,z) = (4,-5,3) + t(0,0,1)$

$\Rightarrow s: \begin{cases} x = 4 \\ y = -5 + t \\ z = 3 \end{cases}$ $\qquad\qquad$ $\Rightarrow p: \begin{cases} x = 4 \\ y = -5 \\ z = 3 + t \end{cases}$

21) a) $r_1: \begin{cases} x = -2 - t \\ y = t \\ z = 3 - 2t \end{cases}$ $\qquad$ $r_2: \begin{cases} x = 2t \\ y = -6 + t \\ z = 1 + t \end{cases}$ $\qquad$ $\vec{v_1} = (-1, 1, -2)$

$\vec{v_2} = (2, 1, 1)$

$\boxed{\cos\theta = \dfrac{|\vec{v_1} \cdot \vec{v_2}|}{|\vec{v_1}| \cdot |\vec{v_2}|}}$ $\qquad$ $\cos\theta = \dfrac{|(-1,1,-2) \cdot (2,1,1)|}{\sqrt{6} \cdot \sqrt{6}}$

$|\vec{v_1}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$ $\qquad$ $\cos\theta = \dfrac{|-2 + 1 - 2|}{6} = \dfrac{3}{6} = \dfrac{1}{2}$

$|\vec{v_2}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

$\cos\theta = \dfrac{1}{2} = 60°$

b) $\eta_1: \begin{cases} y = -2x+3 \\ z = x-2 \end{cases}$ $\eta_2: y = \dfrac{z+1}{-1} ; x = 4$ $\vec{v_2}(1,-1,0)$

$\vec{v_1}(1,-2,1)$

$x=0 ; A(0,3,-2)$

$x=1 ; B(1,1,-1)$ $=$ $\cos\theta = \dfrac{|\vec{v_1} \cdot \vec{v_2}|}{|\vec{v_1}| \cdot |\vec{v_2}|} = \dfrac{|(1,-2,1) \cdot (1,-1,0)|}{\sqrt{6} \cdot \sqrt{2}}$

$\vec{v_1} = (1,-2,1)$

$= \dfrac{1+2}{\sqrt{12}} = \dfrac{3}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2} = 30°$

$|\vec{v_1}| = \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}$

$|\vec{v_2}| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

c) $\eta_1: \begin{cases} x = 1+\sqrt{2}t \\ y = t \\ z = 5-3t \end{cases}$ $\eta_2: \begin{cases} x=3 \\ y=2 \end{cases}$ $\vec{v_1} = (\sqrt{2},1,-3)$

$\vec{v_2} = (0,0,1)$

$|\vec{v_1}| = \sqrt{\sqrt{2}^2+1^2+(-3)^2} = \sqrt{12}$

$\cos\theta = \dfrac{|(\sqrt{2},1,-3) \cdot (0,0,1)|}{\sqrt{12}}$ $|\vec{v_2}| = \sqrt{1^2} = 1$

$\cos\theta = \dfrac{|-3|}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2} = 30°$

d) $\eta_1: \dfrac{x-4}{2} = \dfrac{y}{-1} = \dfrac{z+1}{-2}$ $\eta_2: \begin{cases} x=1 \\ \dfrac{y}{4} = \dfrac{z-2}{3} \end{cases}$ $\vec{v_1} = (2,-1,-2)$

$\vec{v_2} = (0,4,3)$

$|\vec{v_1}| = \sqrt{2^2+(-1)^2+(-2)^2} = \sqrt{9} = 3$ $\cos\theta = \dfrac{|(2,-1,-2) \cdot (0,4,3)|}{3 \cdot 5}$

$|\vec{v_2}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$\cos\theta = \dfrac{|-10|}{3 \cdot 5} = \dfrac{10}{35} = \dfrac{2}{3} \quad \alpha \approx 48,11°$

22) a) $r_1: \dfrac{x-2}{4} = \dfrac{y}{5} = \dfrac{z}{3}$ $\qquad r_2: \begin{cases} y = nx+5 & x=0; \ A(0,5,-2) \\ z = 2x-2 & x=1; \ B(1, n+5, 0) \end{cases}$

$\vec{v_1} = (4, 5, 3)$

$\vec{v_2} = (1, n, 2)$ $\qquad \cos\theta = 30° \qquad\qquad |\vec{v_1}| = \sqrt{4^2+5^2+(3)^2} = \sqrt{50}$

$$\dfrac{|(4,5,3)\cdot(1,n,2)|}{5\sqrt{2}\cdot\sqrt{n+5}} = \dfrac{\sqrt{3}}{2} \qquad\qquad = 5\sqrt{2}$$

$|\vec{v_2}| = \sqrt{1^2 + n^2 + 2^2} = \sqrt{n^2+5}$

$$\dfrac{10n+20}{5\cdot\sqrt{2n^2+10}} = \dfrac{\sqrt{3}}{2}$$

$$10n+20 = 5\cdot\sqrt{6n^2+30}$$

$$(10n+20)^2 = 5^2\cdot\left(\sqrt{6n^2+30}\right)^2$$

$$100n^2 + 400n + 400 = 25(6n^2+30)$$

$$100n^2 + 400n + 400 = 150n^2 + 750$$

$$50n^2 - 400n + 350 = 0 \qquad\rightarrow\qquad n = \dfrac{8\pm6}{2}$$

$$5n^2 - 40n + 35 = 0$$

$$n^2 - 8n + 7 = 0 \qquad\qquad n' = 7$$

$$\Delta = 64 - 28 = 36 \qquad\qquad n'' = 1$$

b) $r_1: \begin{cases} y = nx-1 \\ z = 2x \end{cases}$ $\qquad r_2: \text{eixo } Oy \qquad\qquad |\vec{v_1}| = 1$

$\vec{v_1} = (0, 1, 0) \qquad\qquad |\vec{v_2}| = \sqrt{n^2+5}$

$x=0; \ A(0, -1, 0)$

$x=1; \ B(1, n-1, 2) \qquad \cos\theta = 30°$

$\vec{v_2} = (1, n, 2)$

$$\dfrac{|(0,1,0)\cdot(1,n,2)|}{\sqrt{n^2+5}} = \dfrac{\sqrt{3}}{2}$$

$$n\cdot2 = \sqrt{3n^2+15}$$

$$(2n)^2 = \left(\sqrt{3n^2+15}\right)^2$$

$$4n^2 = 3n^2 + 15$$

$$n^2 = 15$$

$$n = \pm\sqrt{15}$$

23) a)

$M_1: \begin{cases} x = 2mt - 3 \\ y = 1 + 3t \\ z = -4t \end{cases}$ $M_2: \begin{cases} x = 2y - 1 \\ z = -y + 4 \end{cases}$ $M_1 \perp M_2$ $\vec{v_1} \cdot \vec{v_2} = 0$

$\vec{v_1} = (2m, 3, -4)$

$y = 0; \; A(-1, 0, 4)$
$y = 1; \; B(1, 1, 3)$
$\vec{v_2} = (2, 1, -1)$

$\vec{v_1} \cdot \vec{v_2} = 0$

$(2m, 3, -4) \cdot (2, 1, -1) = 0$

$4m + 3 + 4 = 0$

$4m = -7$

$m = -7/4$

b) $M_1: \begin{cases} y = mx + 3 \\ z = x - 1 \end{cases}$ $M_2:$ reta por $A(1, 0, m)$ e $B(-2, 2m, 2m)$

$M_2: (x, y, z) = (1, 0, m) + t(-3, 2m, m)$

$x = 0; \; A(0, 3, -1)$ $\qquad \vec{v_2} = (-3, 2m, m)$
$x = 1; \; B(1, m+3, 0)$
$\vec{v_1} = (1, m, 1)$

$M_1 \perp M_2$ $\qquad$ $(1, m, 1) \cdot (-3, 2m, m) = 0$ $\qquad m = \dfrac{-1 \pm 5}{4}$

$\vec{v_1} \cdot \vec{v_2} = 0$ $\qquad -3 + 2m^2 + m = 0$

$\Delta = 1 + 24 = 25$ $\qquad m' = -6/4 = -3/2$

$m'' = 1$

24) a) $A(3, 2, -1)$ $\qquad M_1: \begin{cases} x = 3 \\ y = -1 \end{cases}$ $M_2: \begin{cases} y = x - 3 \\ z = -2x + 3 \end{cases}$

$\vec{v_1} = (0, 0, 1)$

$x = 0; \; C(0, -3, 3)$
$x = 1; \; D(1, -2, 1)$
$\vec{v_2} = (1, 1, -2)$

$$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 1 & -2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 0 \\ 1 & 1 \end{matrix} = 1\hat{j} - 1\hat{i} \Rightarrow \vec{V} = (-1,1,0)$$

Equações Paramétricas

que passa pelo ponto A.

$$r: (x,y,z) = (3,2,-1) + t(-1,1,0)$$

$$\begin{cases} x = 3 + t \\ y = 2 + t \\ z = -1 \end{cases}$$

b) $A(0,0,0)$ $\qquad r_1: \dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z-3}{2}$ $\qquad r_2: \begin{cases} x = 3t \\ y = -t + 1 \\ z = 2 \end{cases}$

$$\vec{V_1} = (2,1,2)$$

$$\vec{V_2} = (3,-1,0)$$

$$\vec{V_1} \times \vec{V_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & -1 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 1 \\ 3 & -1 \end{matrix} = 6\hat{j} - 2\hat{k} - 3\hat{k} + 2\hat{i} = 2\hat{i} + 6\hat{j} - 5\hat{k}$$

$$\Rightarrow \vec{V} = (2,6,-5)$$

Equações Paramétricas

que passa pelo ponto A.

$$r: (x,y,z) = (0,0,0) + t(2,6,-5)$$

$$\Rightarrow r \begin{cases} x = 2t \\ y = 6t \\ z = -5t \end{cases}$$

6)

$M_1: \begin{cases} x = 2 + t_1 \\ y = -1 + 2t_1 \\ z = 3t_1 \end{cases}$ $M_2: \begin{cases} x = 1 - t_2 \\ y = t_2 \\ z = 2 + 2t_2 \end{cases}$

I) $y = y$

$t_2 = -1 + 2t_1$

$\vec{v_1} = (1, 2, 3)$ $\vec{v_2} = (-1, 1, 2)$

II) $x = x$

$2 + t_1 = 1 - t_2$

$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ -1 & 1 \end{matrix} = 4-3, -3-2, 1+2$

$2 + t_1 = 1 + 1 - 2t_1$

$\vec{v} = (1, -5, 3)$

$3t_1 = 2 - 3$

Ponto de interseção:

$t_1 = 0$

$M_1: \begin{cases} x = 2 \\ y = -1 \\ z = 0 \end{cases}$ $M_2: \begin{cases} x = 2 \\ y = -1 \\ z = 0 \end{cases}$

$t_2 = -1$

$I(2, -1, 0)$

$\vec{v} = (1, -5, 3)$ Eq. Paramétrica:

$\begin{cases} x = 2 + t \\ y = -1 - 5t \\ z = 3t \end{cases}$

25) a)

$M_1: \begin{cases} y = 2x - 3 \\ z = -x + 5 \end{cases}$ $M_2: \begin{cases} y = -3x + 7 \\ z = x + 1 \end{cases}$

$\Rightarrow M_1: \begin{cases} x = t \\ y = -3 + 2t \\ z = 5 - t \end{cases}$ $\Rightarrow M_2: \begin{cases} x = t \\ y = 7 - 3t \\ z = 1 + t \end{cases}$

$-3 + 2t = 7 - 3t$ $5 - t = t + 1$ $t = t$ $I(2, 1, 3)$

$5t = 10$ $2t = 4$ $2 = 2$

$t = 2$ $t = 2$

b)

$M_1: \begin{cases} 3 + 2t_1 \\ -1 - 3t_1 \\ 2 + 4t_1 \end{cases}$ $M_2: \begin{cases} -1 + t_2 \\ 4 - t_2 \\ -8 + 3t_2 \end{cases}$

I) $3 + 2t_1 = -1 + t_2$ II) $-1 - 3t_1 = 4 - 2t_1 - 4$

$t_2 = 2t_1 + 4$ $-1 = t_1$

$t_2 = 2$

$I(1, 2, -2)$

c) $r_1: \begin{cases} y = 2x - 3 \\ z = -x - 10 \end{cases}$ $r_2: \; x = \dfrac{y-4}{3} = \dfrac{z+1}{-2}$

$\Rightarrow r_1: \begin{cases} x = t \\ y = -3 + 2t \\ z = -10 - t \end{cases}$ $\Rightarrow r_2: \begin{cases} x = t \\ y = 4 + 3t \\ z = -1 - 2t \end{cases}$

I) $t = t$ II) $-3 + 2t = 4 + 3t$ III) $-10 - t = -1 - 2t$

                          $t = -7$ $t = -9$

Não são concorrentes.

d) $r_1: \begin{cases} x = 2 - t \\ y = 3 - 5t \\ z = 6 - 6t \end{cases}$ $r_2: \begin{cases} -3 + 6h \\ y = 1 + 7h \\ z = -1 + 13h \end{cases}$ I) $2 - t = -3 + 6h$

                                                II) $3 - 5t = 1 + 7h$

                                                III) $6 - 6t = -1 + 13h$

I) $t = 5 - 6h$ II) $3 - 25 + 30h = 1 + 7h$ III) $6 - 30 + 36h = -1 + 13h$

                               $23h = 23$ $23h = 23$

$t = -1$ $h = 1$ $h = 1$

$I(3, 8, 12)$

e) $r_1: \begin{cases} x = 2 + t \\ y = 4 - 2t \\ z = 1 + 3t \end{cases}$ $r_2: \begin{cases} x = -1 + 4t \\ y = 2 + 3t \\ z = 5 - 2t \end{cases}$ I) $2 + t = -1 + 4t$

                                                    $3t = 3$

                                                    $t = 1$

II) $4 - 2t = 2 + 3t$ III) $1 + 3t = 5 - 2t$ Não são concorrentes.

   $5t = 2$ $5t = 4$

   $t = 2/5$ $t = 4/5$

$\ell)$

$\eta_1: \begin{cases} x = 2+t \\ y = 4-t \\ z = -t \end{cases}$
$\qquad \eta_2: \begin{cases} y = 6-x \\ z = 2-x \end{cases}$
$\qquad \eta_2: \begin{cases} x = t \\ y = 6-t \\ z = 2-t \end{cases}$

$I)\ 2+t = t \qquad\qquad II)\ 4-t = 6-t \qquad\qquad III)\ -t = 2-t$

Não são concorrentes.

$26)\ a)$
$\bar{5}.$

$\eta_1: \begin{cases} x = t_1 \\ y = -5+2t_1 \\ z = 2-t_1 \end{cases}$
$\qquad \eta_2: \begin{cases} x = 5+t_2 \\ y = mt_2 \\ z = -1+t_2 \end{cases}$
$\qquad\qquad I)\ t_1 = 5+t_2$
$\qquad\qquad II)\ 2-5-t_2 = -1+t_2$
$\qquad\qquad\qquad 2t_2 = -2$
$\qquad\qquad\qquad t_2 = -1$
$\qquad\qquad\qquad t_1 = 4.$
$\qquad\qquad\qquad \bar{5}.$

$III)\ mt_2 = -5+2t_1$
$\qquad -m = -5+8$
$\qquad -m = 3$
$\qquad \underline{m = -3}$

$b)$

$\eta_1: \begin{cases} x = m-t_1 \\ y = 1+t_1 \\ z = 2t_1 \end{cases}$
$\qquad \begin{cases} x = 1+3t_2 \\ y = -2+t_2 \\ z = -2t_2 \end{cases}$

$I)\ 2t_1 = -2t_2 \qquad II)\ 1+t_1 = -2+t_2 \qquad III)\ m+3/2 = 1+9/2$
$\quad t_1 = -t_2 \qquad\qquad 1-t_2 = -2+t_2 \qquad\qquad m = 1+6/2$
$\qquad\qquad\qquad\qquad 2t_2 = 3 \qquad\qquad\qquad m = 8/2$
$\qquad\qquad\qquad\qquad t_2 = 3/2 \qquad\qquad\qquad \underline{m = 4}$
$\qquad\qquad\qquad\qquad \underline{t_1 = -3/2}$

$\dfrac{27)}{3.}$

$M_1: \begin{cases} x = 1 + 2t_1 \\ y = -t_1 \\ z = 3 \end{cases}$
$M_2: \begin{cases} x = t_2 \\ y = -1 + t_2 \\ z = 2 + t_2 \end{cases}$
I) $-t_1 = -1 + t_2$

$t_1 = 1 - t_2$

II) $3 = 2 + t_2$ III) $t_1 = 0$ $I(1, 0, 3)$

$t_2 = 1$ $A(0, 1, 0)$

IV) $P = A + t(I - A)$

$(x, y, z) = (0, 1, 0) + t \cdot (1, -1, 3)$

$r: \begin{cases} x = t \\ y = 1 - t \\ z = 8t \end{cases}$
$\dfrac{x}{1} = \dfrac{y - 1}{-1} = \dfrac{z - 3}{3}$

$\begin{cases} y = -x + 1 \\ z = 3x \end{cases}$

$\dfrac{28)}{3.}$

$r: \begin{cases} x = 2 + t \\ y = t \\ z = -1 + 2t \end{cases}$

$A(2, -1, 2)$ B
$(1, 0, -1)$
d d
$C(2 + t, t, -1 + 2t)$

I) $\vec{AC} = C - A = (t, t + 1, 2t + 1)$ II) $|\vec{AC}| = |\vec{BC}|$

$\vec{BC} = C - B = (t + 1, t, 2t)$

$\sqrt{t^2 + (t+1)^2 + (2t+1)^2} = \sqrt{(t+1)^2 + t^2 + (2t)^2}$

$t^2 + t^2 + 2t + 1 + 4t^2 + 4t + 1 = t^2 + 2t + 1 + t^2 + 4t^2$

$4t = -1$

$t = 1/4$

III) $x = 2 + (-1/4) = 3/4$

$y = -1/4$ $C(3/4, -1/4, -3/2)$

$z = -1 - 2/4 = -3/2$

29)

$$r: \begin{cases} x = 2+t \\ y = 1+2t \\ z = 3+2t \end{cases}$$

a) $\vec{AP} = P - A = (t, 2t, 2t)$

$A(2,1,3)$

I) $|\vec{AP}| = 6$

$$\sqrt{t^2 + (2t)^2 + (2t)^2} = 6$$

$$t^2 + 4t^2 + 4t^2 = 6^2$$

$$t^2 = 36/9 \Rightarrow t = \pm 2$$

II) $P(4,5,7)$ ou

$P(0,-3,-1)$

b) $\vec{BP} = P - B = (1+t, 2+2t, 2t)$

$B(1,-1,3)$

I) $|\vec{BP}| = 2$

$$\sqrt{(1+t^2) + (2+2t)^2 + (2t)^2} = 2 \qquad \Rightarrow t = \frac{-10 \pm 8}{18}$$

$$1 + 2t + t^2 + 4 + 8t + 4t^2 + 4t^2 = 4$$

$$9t^2 + 10t + 1 = 0 \qquad\qquad t' = -1$$

$$\Delta = 100 - 36 \qquad\qquad t'' = 1/9$$

$$\Delta = 64$$

II) $P(1,-1,1)$ ou $P(17/9, 7/9, 25/9)$

30) $A(1,3,5)$ $\qquad$ r: $(x,y,z) = (1,3,5) + t(-1,-3,0)$

$P(0,0,5)$ $\qquad\qquad \Rightarrow$ r: $\dfrac{x-1}{-1} = \dfrac{y-3}{-3}$; $z = 5$

$\vec{AP} = P - A = (-1,-3,0)$

I) $-3x + 3 = -y + 3 \qquad \Rightarrow$ r: $\begin{cases} y = 3x \\ z = 5 \end{cases}$

$\qquad\qquad y = 3x$

31) a) $A(4,-2,2)$ $\qquad$ r: $x = 2y = -2z \Rightarrow$ r: $\dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z}{-1}$

s: $\dfrac{x-4}{2} = \dfrac{y+2}{1} = \dfrac{z-2}{-1}$ $\qquad\qquad \vec{v} = (2,1,-1)$

$\Rightarrow$ s: $\begin{cases} x = -2z + 8 \\ y = -z \end{cases}$

b) $P(0,0,0)$ I) $r: \dfrac{2x-1}{3} = \dfrac{y+2}{-2} = 2z-2$

$r: \dfrac{x-1/2}{3} = \dfrac{y+2}{-4} = z-1$ $\vec{v_r} = (3, -4, 1)$

II) $s: x = -y = -z$ III) $\vec{v} = \vec{v_r} \times \vec{v_s} =$

$\vec{v_s} = (1, -1, -1)$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -4 & 1 \\ 1 & -1 & -1 \end{vmatrix}\begin{matrix} \hat{\imath} & \hat{\jmath} \\ 3 & -4 \\ 1 & -1 \end{matrix}$$

$4\hat{\imath} + \hat{\jmath} - 3\hat{k} + 4\hat{k} + \hat{\imath} + 3\hat{\jmath}$

$5\hat{\imath} + 4\hat{\jmath} + \hat{k}$

$(5, 4, 1)$

IV) $g: \begin{cases} 5t \\ 4t \\ t \end{cases}$

32) $\vec{v} = \vec{AB} = B - A = (-2, 4, -2)$

$\vec{P} = (-2, 4, 0)$

$\cos\theta = \dfrac{|\vec{v} \cdot \vec{P}|}{|\vec{v}| \cdot |\vec{P}|} = \dfrac{|4+16|}{\sqrt{24} \cdot \sqrt{20}} = \dfrac{20}{4\sqrt{30}} = \dfrac{5}{\sqrt{30}}$

$\cos\theta = \dfrac{5\sqrt{30}}{30} = \dfrac{\sqrt{30}}{6} \rightarrow \theta \cong 24,1°$

33) $r \begin{cases} x = t \\ y = 5t-7 \\ z = -2t+6 \end{cases}$ $\vec{v} = (1, 5, -2)$ I) $z = 0$ II) $y = 5x-7$

$\vec{P} = (1, 5, 0)$ $-2x+6=0$ $y = 15-7$

     $x=3$ $y=8$

III) $s: \begin{cases} x = 3+t \\ y = 8+5t \\ z = 0 \end{cases}$

$\dfrac{34)}{\xi}$ a) $A(3,4,-2)$

$P(1+t, 2-t, 4+2t)$

$\vec{v_p} = (1,-1,2)$
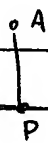
$\vec{v_p} \cdot \vec{PA} = 0$ $PA(4,0,-2)$

$(1,-1,2) \cdot (2-t, 2+t, -2t-6) = 0$

$2-t +(-2-t) + (-4t-12) = 0$

$-6t = 12$

$t = -2$

$s: \begin{cases} x = 3 + 4a \\ y = 4 \\ z = -2 - 2a \end{cases}$

b) $|\vec{AP}| = \sqrt{4^2 + (-2)^2} = \sqrt{20}$ //

c) ?

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